Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

On the Laminar Two-Dimensional Wake of an Incompressible Pseudoplastic Fluid

Chang-yu Liu
University of Singapore, Singapore

Introduction

OWING to the growing importance of the phenomenon attending the flow of pseudoplastic fluid, theoretical as well as experimental studies on this subject have been well documented by many investigators. This Note describes a closed-form solution to laminar boundary-layer equations of a non-Newtonian pseudoplastic fluid in the wake of a flat plate at zero incidence. A velocity distribution function for the value of the parameter N = 0.5 is calculated. It is found that the spread is larger than that for a Newtonian fluid.

Analysis

Under the assumptions of steady and two-dimensional laminar flow, the momentum equation of pseudoplastic fluid can be expressed as

$$u\partial u/\partial x + v\partial u/\partial y = KN\partial/\partial y\,(\,|\partial u/8Y|^{N-1}\partial u/\partial y \qquad (1)$$

where x and y are, respectively, the coordinates along and normal to the direction of the plate, u and v are their corresponding velocities, K and N are parameters related to pseudoplastic fluid. Assuming that the velocity difference in the wake

$$u_I = U - u \qquad (2)$$

is small compared with the freestream velocity U and that higher-order terms in u_I may be neglected, then the far wake momentum equation can be transformed into the form

$$U\frac{\partial u_I}{\partial x} = KN\frac{\partial}{\partial y}\left(\left|-\frac{\partial u_I}{\partial y}\right|^{N-1}\frac{\partial u}{\partial y}\right) \qquad (3)$$

and the boundary conditions are

$$\text{at } y=0, \frac{\partial u_I}{\partial y}=0, \text{ at } y=\infty,\ u_I=0$$

By introducing the following variables

$$u_I = U\alpha\left(\frac{x}{\ell}\right)^{-1/2N} g(\eta)$$

$$\eta = \left(\frac{U^{2-N}\ell^N}{KN}\right)^{1/N+1}\left(\frac{x}{\ell}\right)^{-1/2N}\ \frac{y}{\ell}$$

Where ℓ is the length of the plate, and α is a positive free constant to be defined later, we can transform Eq. (3) into a second-order total differential equation which is given as

$$-1/2N(g+\eta g') = |-\alpha g'|^{N-1}g'' \qquad (4)$$

with the boundary conditions

$$g'=0 \text{ at } \eta=0,\ g=0 \text{ at } \eta=\infty$$

Integrating Eq. (4) once and employing the boundary condition $g'=0$ at $\eta=0$, we have

$$\tfrac{1}{2}\eta g = \alpha^{N-1}|-g'|^N \qquad (5)$$

When $N=1$, Eq. (5) reduces to the one for Newtonian fluid

$$g' + \tfrac{1}{2}\eta g = 0$$

Integrating Eq. (5) again and assuming that g' is negative ($\eta > 0$) gives

$$g = \left[1-\frac{N-1}{N+1}\left(\frac{1}{{}_{2\alpha}N-1}\right)^{1/N}|\eta|N+1/N^{N/N-1}\right] \qquad (6)$$

where, following Ref. 1, we have assumed that $g=1$ at $\eta=0$. Also, the absolute value of η is used owing to the nature of the wake which is symmetric with respect to the line $\eta=0$. (The same scheme has been applied by Schlichting[2] for the case of a two-dimensional turbulent wake.) It can be easily shown that the boundary condition $\eta=\infty$, $g=0$ can also be satisfied for $N<1$. Thus the drag of the plate may be expressed as

$$D = \rho bU\int_{-\infty}^{\infty} u_I dy = \rho bU^2\left(\frac{U^{2-N}\ell^N}{KN}\right)^{-1/N+1}$$

$$\ell\alpha\int_{-\infty}^{\infty}\left[1-\frac{N-1}{N+1}\left(\frac{1}{{}_{2\alpha}N-1}\right)^{1/N}|\eta|\right]^{N/N-1}d\eta \qquad (7)$$

By using Acrivos, Shah, and Peterson's result[3] for the local shearing stress at the wall

$$\tau_w = \rho U^2\left[\frac{U^{2-N}x^N}{KN}\right]^{-1/N+1} \qquad (8)$$

we can calculate the frictional force on the plate wetted on both sides in the form

$$D = 2\rho bU^2C\left[\frac{U^{2-N}\ell^N}{KN}\right]^{1/N+1}\ell(N+1) \qquad (9)$$

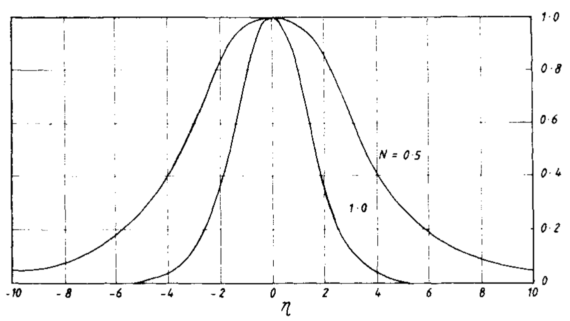

Fig. 1 Velocity profiles for $N=0.5$ and 1.0.

Received March 30, 1976; revision received Aug. 11, 1976.
Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.
*Associate Professor, Mechanical and Production Engineering Department.

where C is a constant depending on N only[3] and b is the breadth of the plate. Equating Eqs. (7) and (9) yields

$$2(N+1)C=\alpha\int_{-\infty}^{\infty}\left[1 - \frac{N-1}{N+1}\left(\frac{1}{2C^{N-1}}\right)^{1/N}|\eta|N+1/N\right]^{N/N-1}d\eta \qquad (10)$$

For any value of N, C can be obtained from Ref. 3, and hence α can be found from Eq. (10). By substituting the value of α into Eq. (6), the velocity function g can then be obtained. For the case of $N=0.5$, $C=0.58$, Eq. (10) may be simplified to

$$3\times 0.58=d\eta/1-\alpha/12|\eta|^3 \qquad (11)$$

Assuming that $\eta_1=(\alpha/12)^{1/3}\eta$, Eq. (11) becomes

$$3\times 0.58=(12)^{1/3}\alpha^{2/3}\int_{-\alpha}^{\alpha}d\eta_1/1-|\eta_1|^3=(12)^{1/3}\alpha^{2/3}\pi/\sqrt{3}$$

and therefore α is equal to 0.271. From Eq. (6), the velocity function g becomes

$$g=1/1+0.0226|\eta|^3 \qquad (12)$$

g is plotted vs η as shown in Fig. 1. The special case of $N=1$ is also shown.

References

[1] Atkinson, C., "On the Laminar Two-Dimensional Free Jet of an Incompressible Pseudoplastic Fluid," *Journal of Applied Mechanics*, Vol. 39, Dec. 1972, pp. 1162-1164.

[2] Schlichting, H., *Boundary Layer Theory*, 6th ed., McGraw-Hill, New York, 1968, pp. 691-695.

[3] Acrivos, A., Shah, M. J., and Peterson, E. E., "Momentum and Heat Transfer in Laminar Boundary Layer Flow of non-Newtonian Fluids Past External Surface," *AIChE Journal*, Vol. 4, 1960, p. 312.

Added Mass of a Rectangular Cylinder in a Rectangular Canal

Kwang June Bai*

David W. Taylor Naval Ship Research and Development Center, Bethesda, Md.

Introduction

IN this Note the added mass of a rectangular cylinder in a rectangular canal is considered as the frequency tends to zero, i.e., with a rigid free-surface boundary condition. For small clearances between the body and the canal, an approximate formula for the added mass is obtained from simple elementary solutions in subdivided finite elements. The sway added mass of the rectangular cylinder with half-beam a and draft b located at the center of a rectangular canal with half-width c and depth h, for the zero-frequency limit, is given by

$$\mu=\frac{m}{3a}\left\{\frac{bh}{c-a}+\frac{b(2a+c)}{h-b}+(c-a)\right\}$$

where μ is the sway added mass for the zero-frequency limit and m is the mass of the fluid displaced by the body.

To test the validity of the previous formula, several comparisons are made with results obtained by the straightforward finite-element method based on the dual extremum principles of Bai.[1,2] This formula is shown to provide good predictions of added mass for the range

$$0<(h-b)/a)\leq 0.3, \qquad 0<(c-a)/b\leq 0.3$$

The formula is also compared with a formula obtained by Newman.[3]

Received June 7, 1976; revision received Aug. 20, 1976.

Index categories: Wave Motions and Sloshing; Marine Hydrodynamics.

*Naval Architect, Hydrodynamics Branch.

Mathematical Formulation and Solution

The computation of the added mass of a cylinder in a canal or restricted water has practical importance in naval hydrodynamics, e.g., in ship maneuvering. There are several methods to compute the added mass: the method of conformal mapping, the method of Green's function, the finite-element method, or the hypercircle method, etc.

Recently the added mass in water of finite depth has been computed by the finite-element method based on the dual extremum principles of Bai.[4] Fujino[5-8] has investigated the added mass of two-dimensional cylinders in a canal for the limiting frequencies using the hypercircle method first advocated by Synge. Bai[1,2] also computed the added mass of triangular, rectangular, circular, and Lewis-form sections in a canal by the finite-element method based on the dual extremum principles. Newman[9] obtained analytic expressions for the added mass in a canal. Recently, Newman[3] also obtained an approximate formula for the added mass of a rectangular cylinder by assuming that the clearance between the body and the canal is small.

In this Note we present an approximate formula of the added mass in a canal under assumptions similar to those made by Newman.[3] By making use of a simple solution of Laplace's equation, an approxmate formula for the added mass is obtained in terms of the geometry of the body and canal for small clearances between the body and the canal walls. In spite of the extremely simple way of obtaining the approximate formula, this formula gives good approximate results for a range of small body-canal clearances.

The sway added mass μ in the zero-frequency limit can be interpreted as the zero Froude-number limit for horizontal translatory motion. This can also be interpreted as the heave added mass in the infinite frequency limit by rotating all of the boundary configurations by 90° when the geometry is symmetric with respect to the y axis. However, the zero limiting-frequency problem is ill-posed when the floating body is heaving in restricted water.

When we consider the computation of the sway added mass of a rectangular cylinder in a canal as shown in Fig. 1, we have the following potential-theory formulation in the zero-frequency limit

$$\nabla^2\phi(x,y)=0 \qquad \text{in } R \qquad (1a)$$

$$\phi_n=V\cdot n=-n_1 \qquad \text{on } S_0 \qquad (1b)$$

$$\phi_n=0 \qquad \text{on } S_F\cup S_w\cup S_B \qquad (1c)$$